On Random Walk Hypothesis

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ABSTRACT. The random walk hypothesis popularized by Burton Malkiel's book, A Random Walk Down Wall Street, has been the object of discussions to many scholars and finance professionals. The hypothesis states that short-term stock price movements are statistically random and thus not predictable. The theory is consistent with the efficient-market hypothesis and has real implications for investors and security analysts as it implies the limitations of technical and fundamental analysis to earn a profit in the stock market. This paper will first describe two important approaches to predicting stock price changes and some basic assumptions behind the random walk theory. Then it will focus on examining the independence and the distribution of stock price changes. So far, empirical data shows strong support for the random walk hypothesis.

The random walk hypothesis has elicited many debates and discussions among scholars and finance professionals. This concept was first proposed by the French broker Jules Regnault who published the first book about the modern theory of stock price changes. Formal research can be traced back to the French mathematician Louis Bachelier in 1900. It was not until the 1970s that the term was popularized by the book, *Random Walk Down the Wall Street*, by Burton Malkiel, who was an economics professor at Princeton University. In Malkiel's book, he described an experiment in which he asked his student to construct a stock chart of a hypothetical stock initially selling at $50. Each successive price change was determined by a coin flip. Consequentially, the hypothetical stock chart resembled a normal stock chart and even displayed repetitive patterns.[1]

The random walk hypothesis states that stock price changes are random and are thus cannot be predicted based on past information. The repetitive patterns in the stock market are statistical illusions rather than true patterns. In statistical terms, the theory

[1] Burton Gordon Malkiel, *A Random Walk down Wall Street: The Time-Tested Strategy for Successful Investing* (New York, NY: W.W. Norton & Company, 2020).

implies that successive price changes should be independent, identically distributed random variables. This paper will first explore the theories underlying the random walk hypothesis and then examine the mathematical models in more detail.

Investors and finance professionals persistently aim to predict the future course of the stock market. However, scholars have questioned to what extent past patterns can make meaningful predictions about future stock price movements. The random walk hypothesis is one theory that casts doubt on various stock analysis methods used by investors to predict stock prices. If the hypothesis is true, then stock analysis would become meaningless. In general, there are two common approaches to predicting future stock prices, technical analysis, and fundamental analysis.

The basic assumption of technical analysis is that past patterns in individual securities tend to recur in the future. By studying and interpreting stock charts, technical analysts, often called "technicians", attempt to familiarize themselves with past patterns and identify trends that are likely to recur.[2] That is, future price change is dependent on the sequence of past price changes. However, there are quite many arguments against technical theory. One is that technicians only buy after patterns have been displayed and sell after they are interrupted.[3] These technicians can miss the best timing as stock price shifts often happen of a sudden. Moreover, the value of technical analysis decreases as more and more analysts use this technique.

The fundamental analysis cares little about past patterns of an individual stock. The assumption is that an individual stock has an intrinsic value, which is related to a company's assets, earnings, dividends, and risk. By studying these fundamental factors, the analysts can determine an estimate of a security's intrinsic value.[4] The investors are then advised to buy if the intrinsic value is above the current stock price. In reality, there are two potential drawbacks to this technique. First is that the information from the company or the analyst's evaluation itself might be wrong. Second is that the stock price

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[2] Eugene F. Fama, "Random Walks in Stock Market Prices," *Financial Analysts Journal* 51, no. 1 (1995): pp. 75-80, https://doi.org/10.2469/faj.v51.n1.1861.
[3] Burton Gordon Malkiel, *A Random Walk down Wall Street: The Time-Tested Strategy for Successful Investing* (New York, NY: W.W. Norton & Company, 2020).
[4] Ibd.

may not converge to its intrinsic value in the short run. Despite the downside, about 90 percent of analysts in Wall Street deems themselves as fundamentalists.

Prevalent among finance professionals, technical and fundamental analysis both depend on past records to predict future growth. However, economists and statisticians have taken a radically different approach when analyzing stock prices, which is the random walk hypothesis.[5] Proposed by Eugene Fama, the random walk hypothesis usually starts with the premise that markets are efficient. That is, a large number of rational profit-maximizers compete with each other to predict future prices of individual securities, with important current information available to everyone.[6] The market efficiency theory implies that neither technical nor fundamental analysis can help investors. It is often the case that no one can ever precisely determine the intrinsic value of individual securities due to uncertainty or misinterpretation of available information. Therefore, in an efficient, the buy and sell actions of the competing participants should cause the actual price of a security to wander randomly about its intrinsic value.

In a perfectly efficient market, stock price sequences should be a martingale, while daily prices can fluctuate. The efficient market theory implies that stocks trade at their fair value and investors are impossible to beat the market by certain portfolios or market timing. If people believe the price is going up to $120 tomorrow, the price goes up to $120 today. Suppose X_n represents the closing price at the end of day n of a security. Then,

$$E[X_{n+1}|X_1, X_2, \ldots, X_n] = X_n.$$

If tomorrow's stock price X_{n+1} is expected to increase, then the demand would raise the current price X_n. Similarly, if X_{n+1} is predicted to be lower, competing sellers would enter the market and drive down the current price X_n.[7]

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[5] Bruce D. Fielitz, "On the Random Walk Hypothesis," *The American Economist* 15, no. 1 (1971): pp. 105-107, https://doi.org/10.1177/056943457101500113.
[6] Eugene F. Fama, "Random Walks in Stock Market Prices," *Financial Analysts Journal* 51, no. 1 (1995): pp. 75-80, https://doi.org/10.2469/faj.v51.n1.1861.
[7] Samuel Karlin and Mark A. Pinsky, *An Introduction to Stochastic Modeling* (London: Academic Press, 2011).

The theory of random walk is valid based on two hypotheses. First is that successive price changes in individual security are independent, and second is that the price changes follow a distribution.[8] Of the two hypotheses independence is indispensable for the random walk hypothesis to be valid. Independence simply means that the probability of the price change during time period *t* is independent of the price changes in previous time periods. That is,

$$\mathbb{P}(x_t = x | x_{t-1}, x_{t-2}, \cdots) = \mathbb{P}(x_t = x).$$

Perfect independence may not exist, but the hypothesis is valid as long as the degree of dependence is not sufficient to support that predicting future prices based on history makes more profits than the buy-and-hold strategy. As proposed by Bachelier and then more explicitly by Osborne, “if successive bits of new information arise independently across time, and if noise or uncertainty concerning intrinsic values does not tend to follow any consistent pattern, then successive price changes in a common stock will be independent.”[9] This assumption rather seems extreme since if there is a dependency on noise or new information, it would lead to dependency on successive price changes. For example, opinion leaders in the market can attract followers and change their perceptions about a certain security. However, in an efficient market, if there are many highly astute investors in the market, they would be able to interpret both the “price effects of current new information and of the future information implied by the dependency in the information generating process.”[10] This hypothesis implies that even if the processes generating noise and new information are dependent, stock price changes may also conform to the independence assumption of the random walk hypothesis.

The second hypothesis of the random walk model is that price changes follow some distribution, as the general model does not specify the shape of the distribution.[11] The specification of distribution, however, is very helpful to investors in assessing the risk of common stocks. For instance, the probability of larger fluctuations differs from

[8] Eugene F. Fama, “The Behavior of Stock-Market Prices,” *The Journal of Business* 38, no. 1 (1965): p. 34, https://doi.org/10.1086/294743.
[9] Ibd.
[10] Ibd.
[11] Ibd.

one another even if two different distributions have the same mean or expected price change. Studies have been conducted as early as 1900 to examine stock price changes over fixed durations of a random number of transactions. Bachelier developed the first complete theory of random walk in stock prices. His model was independently derived by Osborne fifty years later. The Bachelier-Osborne model assumes that price changes over successive transactions are independent random variables and have a common finite variance. Then, according to the central limit theorem, the price change of many transactions over a fixed duration should follow a normal, Gaussian distribution. Moreover, the variances of daily, weekly, and monthly distributions are proportional to their durations respectively. For example, if the daily distribution has a variance of σ^2, then the variance of weekly distribution should be approximately $5\sigma^2$.[12]

However, empirical studies could not provide adequate evidence for the Gaussian model, including Osborne's work. Scholars discovered that the distribution of price changes is leptokurtic, meaning that extremely small and large price changes are appearing more frequently than indicated by the normal distribution. The following calculation can demonstrate the discrepancies between the normal distribution and the observed distribution. Let M be the daily price change defined by the difference between the closing price on one trading day and the closing price on the next. Then,

$$M = \xi_0 + \xi_1 + \xi_2 + \cdots + \xi_N = \xi_0 + X,$$

where ξ_i are independent normally distributed random variables with mean 0 and variance of σ^2, and N is the number of transactions in a given trading day that follows a Poisson distribution with mean λ. Suppose $N = n$, and $M = \xi_0 + \xi_1 + \xi_2 + \cdots + \xi_n$ has mean 0 and variance $(n+1)\sigma^2$. The conditional density function is

$$\phi(m) = \frac{1}{\sqrt{2\pi(n+1)}\sigma} e^{\left(-\frac{m^2}{2(n+1)\sigma^2}\right)}.$$

The probability mass function for the number of daily transactions is

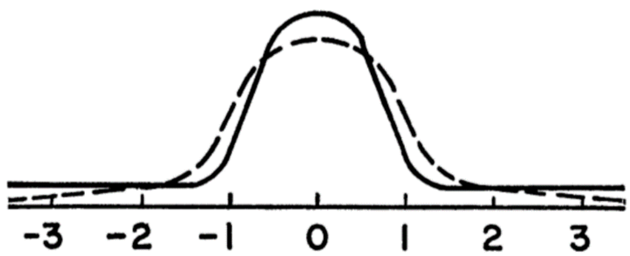
$$p_N(n) = \frac{\lambda^n e^{-n}}{n!}.$$

[12] Eugene F. Fama, "The Behavior of Stock-Market Prices," *The Journal of Business* 38, no. 1 (1965): p. 34, https://doi.org/10.1086/294743.

Therefore, the probability of density function for daily price change is

$$f_M(m) = \sum_{n=0}^{\infty} \phi_M(m) \frac{\lambda^n e^{-\lambda}}{n!}.$$ [13]

When $\lambda = 1$ and $\sigma^2 = \frac{1}{2}$, the random sum distribution has a variance of 1 but shows a consistent departure from normality. The figure below provides a better illustration of the comparison.



Source: Eugene F. Fama, "The Behavior of Stock-Market Prices"

The dashed curve represents the standard normal distribution and the solid curve represents the random sum distribution. An excess of observations exists within 0.5 standard deviations of the mean as the solid curve of the observed distribution is above the dashed curve for the normal distribution. However, the solid curve runs below the dashed curve between 0.5 and 1.0 standard deviation. Therefore, the frequency of values that are between 0.5 and 1.0 standard deviation away from the mean tends to be lower than the normal distribution.

Benoit Mandelbrot is one of the first scholars who seriously question the Gaussian model that in the past, academic research has ignored the implications of the leptokurtosis usually observed in empirical studies.[14] Mandelbrot states that if there are many extreme values, simply excluding them undermines the significance of the tests using the remainder of the data.

So far, the empirical studies have shown strong support for the random walk hypothesis. In an uncertain world, however, no amount of empirical work is enough to establish or completely demolish the validity of the random walk hypothesis. This

[13] Samuel Karlin and Mark A. Pinsky, *An Introduction to Stochastic Modeling* (London: Academic Press, 2011).
[14] Eugene F. Fama, "The Behavior of Stock-Market Prices," *The Journal of Business* 38, no. 1 (1965): p. 34, https://doi.org/10.1086/294743.

hypothesis implies that investors who spend time and money into studying the history of stock prices might just be as well off to employ the buy-and-hold strategy, since interpreting old information is of no real value. Unless the technicians and fundamental value analysts discover more solid evidence, the validity of the random walk hypothesis persists.

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